BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B70 11014

SUBJECT: Structural Moments in a Rotating Spacecraft
Case 620

DATE: November 6, 1970

FROM: L. E. Voelker

ABSTRACT

A structural loads analysis of a rotating spacecraft with appendages must consider the moments acting at the joints which connect these appendages to the main body of the spacecraft. If a line drawn through an appendage's mass center and its joint does not intersect the axis of rotation perpendicularly, the joint must apply a moment to maintain the relative position of the appendage's center of mass. If the axis of rotation is not parallel to one of the principal axes of the appendage, the joint must apply an additional moment to maintain the misaligned attitude. This additional moment component is called the Euler moment. The exact expression for the total moment is presented here and two examples are investigated. For a typical Skylab solar array, the contribution of the Euler moment to the total could be significant. The deflections in a thin rod resulting from the Euler moment reveal that scientific instruments which require precise alignments could be adversely affected by this moment.

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MEMORANDUM FOR FILE

INTRODUCTION

In an elementary structural loads analysis of a rotating spacecraft with appendages the only moment considered is proportional to the mass of the appendage times the acceleration of its center of mass. This moment is required to maintain the position of the appendage's mass center relative to the spacecraft center of mass and is herein called the centrifugal moment.

However, Euler's equations for steady, torque-free rotation of a rigid body show that the angular velocity vector must be parallel to a principal axis of the body. So, if the angular velocity vector of the spacecraft (and the appendage) is not parallel to a principal axis of the appendage, an additional moment is required to satisfy Euler's equations. This moment is called the Euler moment. Its significance is investigated in this memorandum.

CENTRIFUGAL & EULER MOMENTS

Figure 1 shows a model of a spacecraft composed of a central body and a rigid appendage connected to it at a single joint. If the vehicle is rotating with constant angular velocity, the force acting through the joint is

$$\underline{F} = m\, \underline{\omega} \times (\underline{\omega} \times \underline{r}_{CM})$$

where

m - the mass of the appendage

$\underline{\omega}$ - the angular velocity of the vehicle reference frame (X, Y, Z) with respect to an inertial reference frame

$\underline{r}_{CM}$ - (X_{CM}, Y_{CM}, Z_{CM}) the position vector of the center of mass of the appendage in vehicle coordinates

The centrifugal moment acting at the joint is

$$\underline{M}_C = (\underline{r}_{CM} - \underline{r}_J) \times \underline{F}$$

where

$\underline{r}_J = (X_J, Y_J, Z_J)$ the position vector of the joint of the appendage in vehicle coordinates

If the angular velocity vector is not aligned with a principal axis of the appendage, then the Euler moment acting at the joint is

$$\underline{M}_E = \underline{\omega} \times I \, \underline{\omega}$$

where I is the inertia matrix of the appendage about its center of mass. The total moment acting at the joint is

$$\underline{M}_J = \underline{M}_C + \underline{M}_E$$

$$= m(\underline{r}_{CM} - \underline{r}_J) \times (\underline{\omega} \times (\underline{\omega} \times \underline{r}_{CM})) + \underline{\omega} \times I \, \underline{\omega} \quad .$$

Aligning the coordinate axes x y z of the appendage with the vehicle axes X Y Z so that $\underline{M}_J$ and $\underline{\omega}$ have identical components in either system, we may write out the X- component of moment at the joint as

$$M_{JX} = m(Y_{CM}-Y_J) \; [\omega_x\omega_z \, X_{CM} + \omega_z\omega_y \, Y_{CM} - (\omega_x^2+\omega_y^2) \, Z_{CM}]$$

$$+ \; m(Z_J-Z_{CM}) \; [\omega_x\omega_y \, X_{CM} + \omega_y\omega_z \, Z_{CM} - (\omega_x^2+\omega_z^2) \, Y_{CM}]$$

$$+ \; \omega_x\omega_y \, I_{xz} - \omega_x\omega_z \, I_{xy} + (\omega_y^2-\omega_z^2) \, I_{yz} + \omega_y\omega_z \, (I_{zz}-I_{yy}) .$$

This expression is more rigorously developed in the appendix.

SKYLAB SOLAR ARRAY

A typical Skylab solar array was chosen as a specific example. The principal axes x y z of the solar array are assumed parallel to the spacecraft X Y Z axes. For a configuration without the ATM, the Workshop is nearly symmetric about the X-axis and the solar arrays are symmetrically offset from the Y axis. The axis of maximum principal moment of inertia of this configuration is assumed to lie in the Y -Z plane at an angle θ from the vehicle Z axis. For stable rotation, the angular velocity vector lies on the axis of maximum moment of inertia and may be written

$$\underline{\omega} = \omega\ (0,\ \sin\theta,\ \cos\theta)\ \text{rad/sec}$$

where ω is the rate of rotation. The only component of Euler moment which exists is the X-component. The Euler and centrifugal X-components of moment were studied as functions of θ. The results show that as θ varies from 0° to 45°, the Euler moment M_{EX} contributes 15-18% of the total moment.

DEFLECTIONS OF A THIN ROD

Figure 2a shows a long, slender rod of mass m simply supported at points A and B in a plane parallel to the X-Z plane. The axis of the rod lies at an angle of 45° from the angular velocity vector. The projections of the deflected rod on the X-Y and X-Z planes are shown in Figures 2b, 2c. The deflection in Figure 2b is the result of the force in the Y direction, while the unusual deflection in Figure 2c is associated with the Euler moment. Depending on the orientation of the rod, these two deflected shapes may be separated, as shown, or they may be superimposed on each other.

CONCLUSIONS

The Euler moments in a rotating vehicle may prove to be significant and should be considered in the analysis or design of the vehicle and its components. This moment can also cause unusual deflections in components depending on the orientation of the component in the vehicle. Therefore the design of any scientific instrument requiring precise tolerances should consider this moment if the instrument is to be installed on a rotating vehicle.

L. E. Voelker

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Attachments
Appendix
Figures 1 and 2

APPENDIX

Consider a vehicle with a single appendage as in Figure 1 steadily rotating about its center of mass with angular velocity $\underline{\omega}$ with respect to an inertial reference frame. The XYZ vehicle axis system is parallel to the xyz system of the appendage. The acceleration of any point in the vehicle is

$$\underline{a} = \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

where $\underline{r} = (X, Y, Z)$ is the position vector from the vehicle center of mass. The force distribution in the vehicle is then

$$\underline{f} = \rho \, \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

where ρ (X, Y, Z) is the mass distribution of the vehicle.

The moment at the joint of the appendage, which has a position vector $\underline{r}_J = (X_J, Y_J, Z_J)$, is

$$\underline{M}_J = \int_V \rho \, (\underline{r} - \underline{r}_J) \times (\underline{\omega} \times (\underline{\omega} \times \underline{r}) \, dV$$

where V is the volume of the appendage. Examining the X component of moment only,

$$M_{JX} = \int_V \rho \, (Y - Y_J) \, [\omega_x \omega_z \, X + \omega_z \omega_y \, Y - (\omega_x^{\,2} + \omega_y^{\,2}) Z] \, dV$$
$$+ \int_V \rho \, (Z_J - Z) \, [\omega_x \omega_y \, X + \omega_z \omega_y \, Z - (\omega_x^{\,2} + \omega_z^{\,2}) Y] \, dV$$

Defining a local position vector with origin located at the appendage's center of mass (X_{CM}, Y_{CM}, Z_{CM}), the position vector may be rewritten as

$$(X, Y, Z) = (X_{CM} + x, \; Y_{CM} + y, \; Z_{CM} + z)$$

and the X - component of moment becomes

$$
\begin{aligned}
M_{JX} = & \int_V \rho (Y_{CM} - Y_J) [\omega_x \omega_z X_{CM} + \omega_z \omega_y Y_{CM} - (\omega_x^2 + \omega_y^2) Z_{CM}]\, dV \\
& + \int_V \rho (Z_J - Z_{CM}) [\omega_x \omega_y X_{CM} + \omega_y \omega_z Z_{CM} - (\omega_x^2 + \omega_z^2) Y_{CM}]\, dV \\
& + \int_V \rho (Y_{CM} - Y_J) [\omega_x \omega_z\, x + \omega_z \omega_y\, y - (\omega_x^2 + \omega_y^2)\, z\,]\, dV \\
& + \int_V \rho (Z_J - Z_{CM}) [\omega_x \omega_y\, x + \omega_y \omega_z\, z - (\omega_x^2 + \omega_z^2)\, y\,]\, dV \\
& + \int_V \rho\, y\, [\omega_x \omega_z X_{CM} + \omega_z \omega_y Y_{CM} - (\omega_x^2 + \omega_y^2) Z_{CM}]\, dV \\
& - \int_V \rho\, z\, [\omega_x \omega_y X_{CM} + \omega_y \omega_z Z_{CM} - (\omega_x^2 + \omega_z^2) Y_{CM}]\, dV \\
& + \int_V \rho\, y\, [\omega_x \omega_z\, x + \omega_z \omega_y\, y - (\omega_x^2 + \omega_y^2)\, z\,]\, dV \\
& - \int_V \rho\, z\, [\omega_x \omega_y\, x + \omega_y \omega_z\, z - (\omega_x^2 + \omega_z^2)\, y\,]\, dV
\end{aligned}
$$

In the first two integrals, the only variable in the integrands is $\rho\ (x, y, z)$, the mass distribution of the appendage and $\int_V \rho\, dV = m$, the total mass of the appendage. By the definition of center of mass, $\int_V \rho\, x dV = \int_V \rho\, y\, dV = \int_V \rho\, z dV = 0$, so the following four integrals are identically zero. In the last two integrals we recognize the following elements of the inertia matrix.

$$\int_V \rho xy \; dV = -I_{xy}$$

$$\int_V \rho yz \; dV = -I_{yz}$$

$$\int_V \rho xz \; dV = -I_{xz}$$

$$\int_V \rho (y^2 - z^2) \; dV = \int_V \rho (y^2 + x^2) \; dV - \int_V \rho (x^2 + z^2) \; dV$$

$$= I_{zz} - I_{yy}$$

Then the complete moment component may be written as

$$M_{JX} = m(Y_{CM} - Y_J)[\omega_x\omega_z X_{CM} + \omega_z\omega_y Y_{CM} - (\omega_x^2 + \omega_y^2) Z_{CM}]$$

$$+ m(Z_J - Z_{CM})[\omega_x\omega_y X_{CM} + \omega_y\omega_z Z_{CM} - (\omega_x^2 + \omega_z^2) Y_{CM}]$$

$$- I_{xy}\,\omega_x\omega_z + I_{xz}\,\omega_x\omega_y + (\omega_y^2 - \omega_z^2) I_{yz} + \omega_y\omega_z(I_{zz} - I_{yy}).$$

the M_{JY} and M_{JZ} components may be found by cyclic permutations of X, Y, Z and x, y, z.

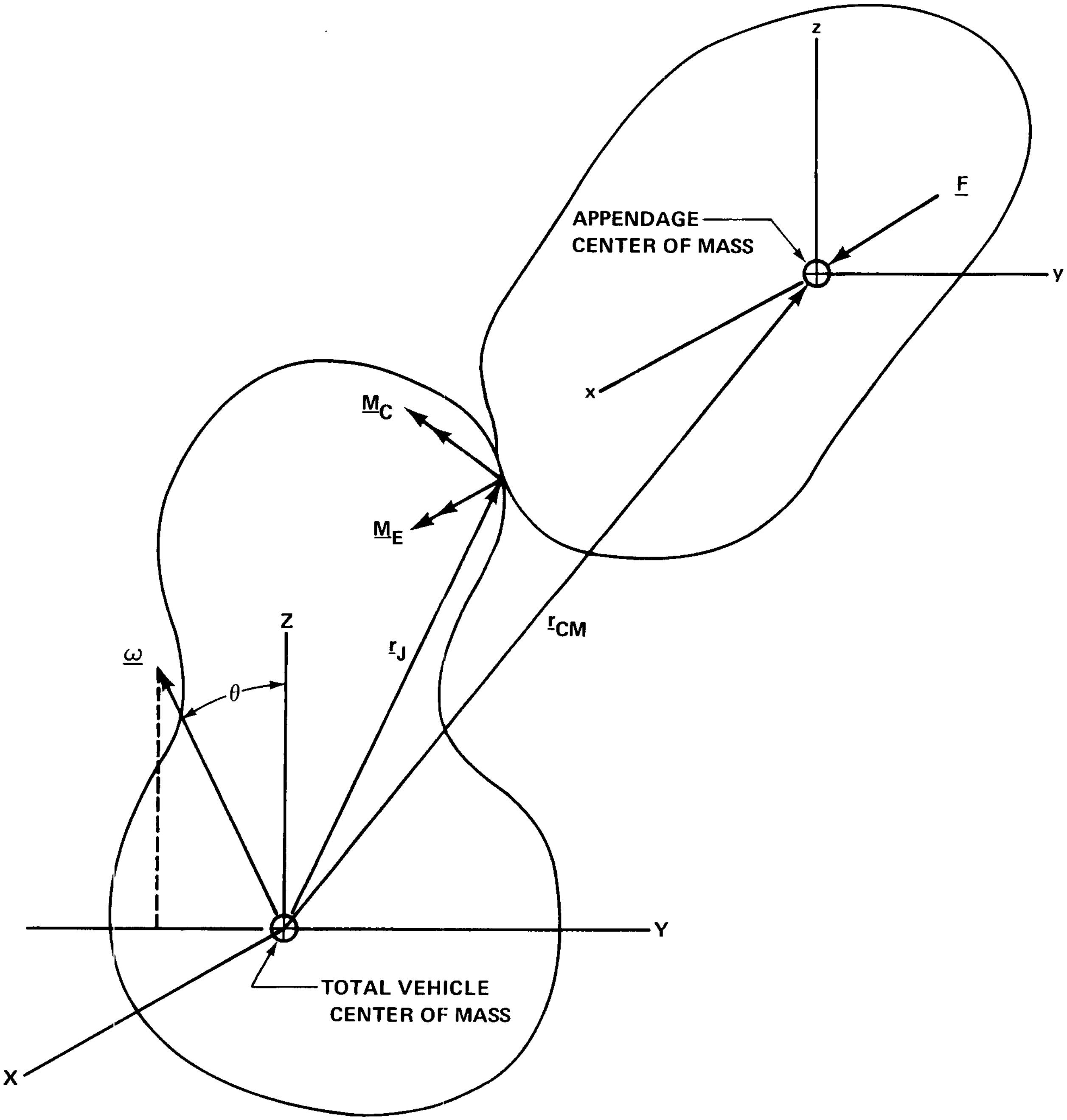

FIGURE 1 - SPINNING SPACECRAFT WITH A SINGLE APPENDAGE

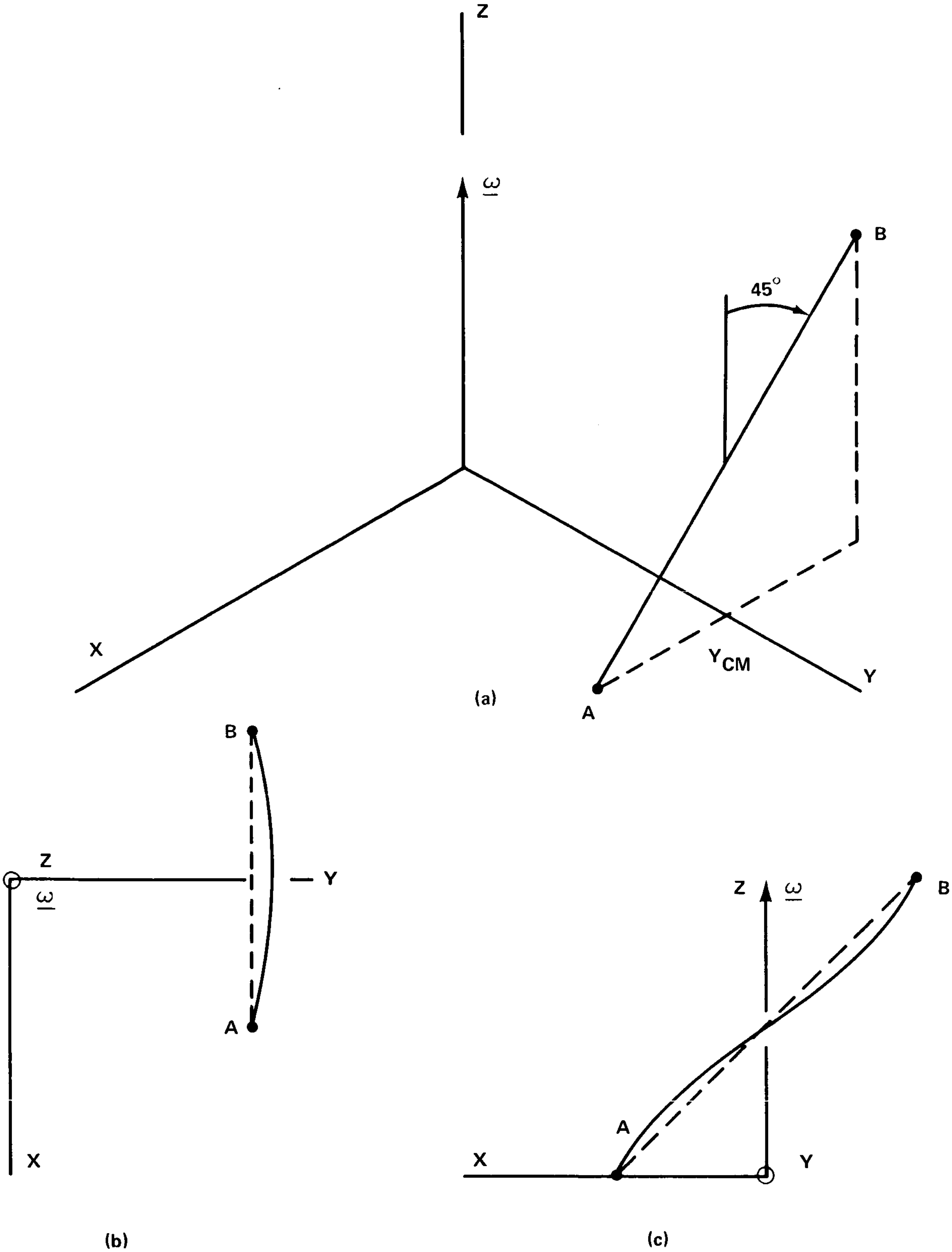

FIGURE 2 - DEFLECTION OF A THIN ROD

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